AD/A-004 775

A MATHEMATICAL MODEL OF HEAD INJURY DUE TO AXISYMMETRIC IMPACT

Han Sun Chan

Aerospace Medical Research Laboratory
Wright-Patterson Air Force Base, Ohio

November 1974

#A004775

NOTICES

When US Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Organizations and individuals receiving announcements or reports via the Aerospace Medical Research Laboratory automatic mailing lists should submit the addressograph plate stamp on the report envelope or refer to the code number when corresponding about change of address or cancellation.

Do not return this copy. Retain or destroy.

Please do not request copies of this report from Aerospace Medical Research Laboratory. Additional copies may be purchased from:

National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

This report has been reviewed and cleared for open publication and/or public release by the appropriate Office of Information (OI) in accordance with AFR 190-17 and DODD 5230.0. There is no objection to unlimited distribution of this report to the public at large, or by DDC to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Henning E. von Gierke

HENNING E. VON GIERKE
Director
Biodynamics and Bionics Division
Aerospace Medical Research Laboratory

AIR FORCE/56780/29 January 1975 — 100

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AMRL-TR-74-27	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD/A-004775
4. TITLE (and Subtitle) A MATHEMATICAL MODEL OF HEAD INJURY DUE TO AXISYMMETRIC IMPACT		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Han Sun Chan		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Aerospace Medical Research Laboratory, Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62202F 72310515
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE November 1974 13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Biological and Medical Sciences
Stress Physiology
Biodynamics
Head Model
Skull Model
Head Injury

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

To demonstrate the mechanisms and provide insight into the dynamics occurring within the human head during short-duration localized impacts, the head has been modeled as a viscoelastic core bonded to a thin viscoelastic spherical shell simulating the brain and the skull, respectively. The potential head injury mechanisms such as cavitation caused by excessive tensile stress and rotation produced by large shear strain, along with their possible damage locations, have been demonstrated by applying an axisymmetric impact over a small area of the shell.

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

Linear viscoelastic properties are assumed for both the core material and the shell. Solution to the second order matric differential equation has been obtained through an iterative method.

PREFACE

The author would like to express his appreciation and gratitude to Dr. H. E. von Gierke and Dr. H. L. Oestreicher for their helpful discussions, support, and guidance. Thanks are due Mr. Ints Kaleps for his generous assistance and helpful suggestions in writing this report. This research was carried out in the Biodynamics and Bionics Division, Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio. At the time the author was in receipt of a resident research associateship from the National Academy of Sciences/ National Research Council.

INTRODUCTION

The special vulnerability of the head to serious or fatal injury is well documented in the publication: "A Survey of Current Head Injury Research" prepared by the Subcommittee on Head Injury of the National Advisory Neurological Diseases and Stroke Council[ref. 1]. Broadly speaking, depending on the state of the skull, i.e., whether or not the skull is penetrated or fractured in the impact process, craniocerebral trauma is termed either open or closed. This investigation is concerned with the injury sustained in closed head impact.

Models of the closed head have been divided in much the same fashion as the major hypotheses proposed to explain closed head injuries due to impact. One group, the rotational school, has contended that the rotational acceleration induced by impact causes high shear strains in the brain matter, rupturing cerebral blood vessels and tissue, while another group, the cavitation school, has claimed that there exist points within the brain where the reduced pressure is sufficient to rupture the capillary walls. Normally, the pressure differential across the capillaries is only a few mmHg. During the impact, the pressure field at a given time contains regions of negative pressure, i.e., less than atmospheric. The transmural pressure at these locations is then suddenly increased, which could lead to the bursting of the capillaries therein. Certainly, when the pressure is reduced to the vapor pressure of the brain substance, cavitation takes place. The catastrophic collapse of the bubbles thus formed is a possible cause of brain damage. An advanced model of the cavitation theory was given by this author[ref. 3]. Based on a suggestion by Goldsmith[ref. 8] and the ground work done by Anzelius[ref. 2], Guttinger[ref. 9], Engin and Liu[ref. 6] modeled the head as a fluid-filled closed spherical shell under torsionless axisymmetric impact loading. The fluid is considered inviscid and represents the cerebrospinal fluid and the brain matter. The shell is elastic, homogeneous, and isotropic and simulates the skull. The wall thickness is thin but includes both the extensional and bending effects. The infinite-series solution was obtained by Engin[ref. 5], through the use of the Laplace transform technique, for the case of an impulsive axisymmetric polar-cap load. In his dissertation[ref. 3], the author has extended the Engin result to include the following additional considerations: (a) An asymmetric impact loading consisting of an axisymmetric pulse coincident with a tangential surface torque (b) A moderately thick shell theory, which includes extensional, bending, rotatory inertia and transverse shear effects. Engin and Roberts[ref. 7] recently examined the problem of the transient response of a fluid-filled elastic spherical shell to an arbitrary velocity input to the shell. The excess pressure distribution in the fluid was evaluated for various deceleration time pulses. As a model for head injury, this problem belongs to the noncontact variety. Since linear deceleration or acceleration of the shell is the only input, the mechanism of injury can only be the excess pressure.

As for models of rotation theory, Lee and Advani, modeled the head as an elastic sphere bonded to a rigid shell undergoing rotational motion. The experimental foundation for this modelling advance is typified by the recent work of Ommaya et al.[ref. 14] and the earlier Holbourn[ref. 11], Owings[ref. 15], working on work of a dissertation supervised by Prof. S. H. Advani, is attacking the problem of the dynamic axisymmetric response of an elastic spherical shell with an elastic core. Hickling and Wenner[ref. 10] modelled the head as a two-layered viscoelastic sphere using the exact equations of linear viscoelasticity. The loadings and motions are considered axisymmetric. The above problems, because of their formulation, are capable of evaluating the relative magnitude and duration of the normal and shear stresses in the elastic core. When the material properties and failure criterion for the brain and skull have been substituted into the model, a tolerance evaluation can be made for a given mechanism of injury.

For any given impact, the head will experience both a translational and rotational acceleration. Even if one were to control the blow in such a manner that the force vector passes through either the center of mass or center of percussion of the head, both motions will occur because of the constraint exerted by the head-neck junction. Implicit in many of the papers of both the translational and rotational theory of brain damage is the impression that translational motions induce only normal stresses or pressure and in rotational motion, only shear stresses. This is not generally true. From continuum mechanics it is known that a normally applied load induces shear stresses in all but the normal directions. Conversely, tangential surface tractions induce normal stresses using a similar argument. Which effect is primary during a given impact is the central question.

It is the intent of this investigation to determine the relative magnitude of these two effects under a given impact. To do so, a more realistic model has to be conceived. The inviscid fluid assumption must give way to include viscosity and a multi-layered ellipsoidal shell should replace the spherical shell. In so doing, the complexities of the boundary conditions together with the nonlinearities in the enclosed flow make a closed-form solution nearly impossible. The approach, which shows the most promise for this analytically intractable problem, is the finite element method.

FORMULATION OF THE PROBLEM

Finite Element Method The traditional method of attacking a physical problem consists in choosing a suitable "infinitesimal" element in the continuum and formulating a differential or integral equation for it. Such an equation, though derived from physical laws, is not itself a physical law; it is a formulation, derived by a certain amount of prior mathematical deduction. The finite element method is a relatively new technique in physical science. It is distinctly different from conventional mathematical or numerical analysis. We cannot say that this type of analysis is inherently better than any other as it does have some limitations, but it also has some distinct advantages.

Finite element analysis suggests that we move the power of numerical analysis forward in the scheme of things, giving it a more direct or even central role in the analytic process. It is essentially a process through which a continuum system with infinite degrees of freedom can be approximated to by an assemblage of subregions or elements each with a specified but finite number of unknowns. In this study the displacement formulation of the finite element method has been applied. We start by dividing the continuum geometrically, in some convenient manner, into a finite number of elements of appropriate size. The elements are assumed to be interconnected at a discrete number of nodal points situated on their boundaries. The displacements of these nodal points are the basic unknown variables of the problem. A set of functions called displacement functions, is chosen to define uniquely the state of displacement within each finite element in terms of its nodal displacement. Each of these elements is then expressed in such a manner that it satisfies the basic laws and the kinematic and constitutive relations, together with certain assumptions and boundary conditions. Whatever changes a system may undergo in a certain interval of time, this method of analysis demands the straight forward criterion that at each instant every individual element must satisfy a balance of mass, momentum, and energy, and must bear pertinent kinematic relationship to its neighboring elements. In fact the finite element method is equivalent to the minimization of the total potential energy of the system in terms of a prescribed displacement field. Its application can be extended to those problems where a variational formulation is possible.

Material Properties and Constitutive Equations The human head is modeled as a solid viscoelastic core bonded to a viscoelastic spherical shell, which simulates the brain matter and the skull, respectively. Linear viscoelastic properties are assumed for both the brain matter and the skull along with homogeneity and isotropy assumptions. In terms of stress matrix $\underline{\sigma}$, strain matrix $\underline{\varepsilon}$, and strain rate matrix $\underline{\dot{\varepsilon}}$, the constitutive equations for the brain matter and the skull for the axisymmetric case in polar coordinate system are in the form:

$$\underline{\sigma} = \underline{D}_1 \underline{\varepsilon} + \underline{D}_2 \underline{\dot{\varepsilon}} \tag{1}$$

where

$$\underline{\sigma} = \begin{bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\phi \\ \tau_{rz} \end{bmatrix}, \quad \underline{\varepsilon} = \begin{bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\phi \\ \gamma_{rz} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_r}{\partial r} \\ \dfrac{\partial u_z}{\partial z} \\ \dfrac{u_r}{r} \\ \frac{1}{2}\left(\dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r}\right) \end{bmatrix} \quad \text{and}$$

$\underline{D}_1$ and $\underline{D}_2$ are 4x4 matrices, which are functions of the material property constants.

Derivation of The Equations of Motion In the displacement formulation of the finite element method, the basic unknown variable are the displacements. The state of displacement $\underline{u}$ in a continuum system is approximated in terms of the nodal displacements $\underline{U}$ by a set of chosen displacement functions $\underline{f}$, i.e.

$$\underline{u} = \underline{f}\underline{U} \tag{2}$$

where

$$\underline{u} = \begin{bmatrix} u_r \\ u_z \end{bmatrix}, \quad \underline{U} = \begin{bmatrix} U_1 \\ U_2 \\ \cdot \\ \cdot \\ \cdot \\ U_n \end{bmatrix} \text{ and}$$

$$\underline{f}(r,z) = \begin{bmatrix} h_1(r,z) & h_2(r,z) & \cdots & h_n(r,z) \\ k_1(r,z) & k_2(r,z) & \cdots & k_n(r,z) \end{bmatrix}$$

Applying the principle of virtual work for dynamic systems, i.e.

$$\delta E = \delta W - \int_v [\delta \underline{u}]^T \rho \, \ddot{\underline{u}} \, dv \tag{3}$$

where E and W are the virtual strain energy and virtual work, respectively, for any compatible virtual displacement $\delta\underline{u}$. The last term in the above equation is the work due to inertial force, and ρ is the density of the material.

Taking first and second derivatives with respect to the independent variable time and spatial variables of equation 2, we have

$$\dot{\underline{u}} = \underline{f}\dot{\underline{U}}, \quad \ddot{\underline{u}} = \underline{f}\ddot{\underline{U}}$$

and

$$\underline{\epsilon} = \underline{g}\underline{U}, \quad \dot{\underline{\epsilon}} = \underline{g}\dot{\underline{U}} \tag{4}$$

where $g = g(r,z)$ is obtained by taking first derivatives with respect to the independent spatial variables of the displacement function matrix $f(r,z)$, specifically,

$$\underline{g} = \begin{bmatrix} \frac{\partial h_1(r,z)}{\partial r} & \frac{\partial h_2(r,z)}{\partial r} & \cdots & \frac{\partial h_n(r,z)}{\partial r} \\ \frac{\partial k_1(r,z)}{\partial z} & \frac{\partial k_2(r,z)}{\partial z} & \cdots & \frac{\partial k_n(r,z)}{\partial z} \\ \frac{h_1(r,z)}{r} & \frac{h_2(r,z)}{r} & \cdots & \frac{h_n(r,z)}{r} \\ \frac{1}{2}\left(\frac{\partial h_1(r,z)}{\partial z} + \frac{\partial k_1(r,z)}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial h_2(r,z)}{\partial z} + \frac{\partial k_2(r,z)}{\partial r}\right) & \cdots & \frac{1}{2}\left(\frac{\partial h_n(r,z)}{\partial z} + \frac{\partial k_n(r,z)}{\partial r}\right) \end{bmatrix}$$

From equations 1 and 4, we obtain

$$\underline{\sigma} = \underline{D}_1 \underline{g} \, \underline{U} + \underline{D}_2 \, \underline{g} \, \dot{\underline{U}} \tag{5}$$

Neglecting the body force, then the virtual strain energy and virtual work due to surface load p, are

$$\mathbf{E} = \int_v \underline{\epsilon}^T \underline{\sigma} \, dv$$

and

$$\mathbf{W} = \int_s \underline{u}^T \underline{p} \, ds \tag{6}$$

Substituting equation 6 into equation 3 and making use of equations 4 and 5, we have

$$\int_v [\delta \underline{U}]^T \underline{g}^T [\underline{D}_1 \underline{g} \, \underline{U} + \underline{D}_2 \underline{g} \, \dot{\underline{U}}] \, dv$$

$$= \int_s [\delta \underline{U}]^T \underline{f}^T \underline{p} \, ds - \int_v [\delta \underline{U}]^T \underline{f}^T \rho \, \underline{f} \, \ddot{\underline{U}} \, dv$$

Since $\delta\underline{U}$ is arbitrary,

$$[\int_v \underline{g}^T \underline{D}_1 \underline{g}\, dv]\, \underline{U} + [\int_v \underline{g}^T \underline{D}_2 \underline{g}\, dv]\, \dot{\underline{U}}$$
$$= \int_s \underline{f}^T \underline{p}\, ds - [\int_v \underline{f}^T \rho\, \underline{f}\, dv]\, \ddot{\underline{U}}$$
$$\underline{M}\,\ddot{\underline{U}} + \underline{C}\,\dot{\underline{U}} + \underline{K}\,\underline{U} = \underline{P} \qquad (7)$$

where

$\underline{M} = \int_v \underline{f}^T \rho\, \underline{f}\, dv$ represents the mass matrix of the equivalent discrete system
$\underline{C} = \int_v \underline{g}^T \underline{D}_2 \underline{g}\, dv$ is the viscous damping matrix
$\underline{K} = \int_v \underline{g}^T \underline{D}_1 \underline{g}\, dv$ is the stiffness matrix, and
$\underline{P} = \int_s \underline{f}^T p\, ds$ is the equivalent concentrated force due to the surface load.

METHOD OF SOLUTION

The governing equations of the dynamic system are expressed by the second order differential equations in matrix form, shown in equation 7, which are essentially a state of equilibrium between the internal and the external forces on each element of the system. Solutions to these equations are accomplished through an iterative procedure to generate the time histories by first determining the instantaneous acceleration over a short time interval for each degree of freedom and then performing integrations to obtain the velocity and displacement changes over the short time interval. These velocity and displacement changes are then added to their previous values to update the time histories. The brief outline for the procedure is as follows: Assuming the acceleration is linear over the small time increment $\Delta t = t_n - t_{n-1}$, then

(i) The velocity at t_n is calculated as

$$\dot{\underline{U}}_{t_n} = \dot{\underline{U}}_{t_{n-1}} + \tfrac{1}{2}(\Delta t)\{\ddot{\underline{U}}_{t_{n-1}} + \ddot{\underline{U}}_{t_n}\}$$

The computation starts with an assumed value of $\ddot{U}_{t_n} = 0$ (or any other reasonable values)

(ii) The displacement at t_n is then computed from

$$\underline{U}_{t_n} = \underline{U}_{t_{n-1}} + \tfrac{1}{2}(\Delta t)\{\dot{\underline{U}}_{t_{n-1}} + \dot{\underline{U}}_{t_n}\} + \tfrac{1}{6}(\Delta t)^2\{\ddot{\underline{U}}_{t_n} - \ddot{\underline{U}}_{t_{n-1}}\}$$

(iii) The error introduced by the assumed values of $\ddot{\underline{U}}_{t_n}$ in step (i) is corrected by re-computing the acceleration $\ddot{\underline{U}}_{t_n}$ from the equations of motion, eq. 7, with the velocity $\dot{\underline{U}}_{t_n}$ and displacement $\underline{U}_{t_n}$ given by (i) and (ii) i.e.

$$M\ddot{\underline{U}}_{t_n} = P_{t_n} - C\dot{\underline{U}}_{t_n} - K\underline{U}_{t_n}$$

with this computed value $\ddot{\underline{U}}_{t_n}$ the calculations go back to steps (i) through (iii) and this procedure is repeated again until convergence has been obtained to the desired accuracy.

NUMERICAL RESULTS AND OBSERVATIONS

The geometric dimensions and the material constants used for the inner solid viscoelastic core, which represents the brain matter, and the outer viscoelastic spherical shell, which simulates the skull, are as follows(ref. 3, 18):

radius	3 in.	
density	0.00116 (slug/in.^{3})	
bulk modulus	3×10^5 lbf/in.^{2}	Core
shear modulus	3.0 lbf/in.^{2}	
viscosity coefficient	0.001 lbf-sec^2/in.^{2}	

[illegible] 3 in
[illegible] .25 in
density [illegible] g/in [illegible]
[illegible] lbf/in [illegible]
[illegible] 0.45
viscosity coefficient 0.009 lbf-sec/in [illegible]

Two [illegible] divis[illegible] for [illegible] system [illegible] figures [illegible] and 2, [illegible] carry out [illegible] As the size of the elements g[illegible] results from these two schemes converge and are in close agreement with [illegible] in this study [illegible] checked in this [illegible] In figures 1 and 2, the z-axis is the [illegible] axis of revolution. [illegible] applied over a small polar angle [illegible] pulse [illegible] durations figure 3, and the exponential time function pulse figure [illegible]

The [illegible] under the rectangular pulse are given in figures 5 through [illegible] The results [illegible] for the rectangular [illegible] with an amplitude of 1000 lb/in² and a duration of [illegible] are for the rectangular pulse with a longer duration of 0.06 msec. [illegible] distribution along the axis of symmetry (the diameter passing through the centroid of the impact area) are plotted successively from time t = 0.02 msec to t = 0.20 msec with an increment of [illegible] By comparing figure 5 with figure 6, the influence of the duration of the applied rectangular pulse is easily observed. Both the amplitude and the pattern of the pressure (stress) field in the core are altered with the increased duration. The pressure histories at the impact pole for pulses of different durations are given in figures 7 and 8. From these and the foregoing figures, it is evident that the increase in pulse length not only increases the amplitude of the pressure field but also increases the duration of the rarefaction, i.e., reduced pressure or tensile stress, in the core. Rarefaction or reduced pressure is the cause of brain damage according to the cavitation hypothesis. The apparent wave speed in the viscoelastic core can be calculated from figures 5 or 6 by observing the arrival time of the wave front at different locations and the distance between these locations. The calculated wave speed is 53,700 in./sec which is very close to the experimental wave speed in brain matter[ref. 19].

In general, the duration of an actual impact on the head is approximately one millisecond or longer and the two rectangular pulses with durations of .01 and .06 msec respectively, as used in figures 7 and 8 are not realistic. They are used here only to demonstrate the effect of the impact duration on the stress (or pressure) field, rather than to simulate a physically obtainable impact. A more meaningful and realistic situation is simulated by a pulse with an exponential time function as shown in figure 4. Results obtained with this loading pulse are presented in figures 9 through 15.

The pressure histories of locations where rarefaction, i.e., reduced or negative pressure, occur are given in figures 9 and 10. All those locations are situated along the axis of symmetry, or the impact diameter (The diameter passing through the centroid of the impact area). These correspond to possible locations of brain damage according to the cavitation hypothesis. The pressure histories for the locations at $z=3.0$ in. (the impact pole), $z=0.12$ in. (close to the center of the sphere), $z=-1.67$ in., and $z=-3.0$ in. (the opposite pole), are given in figures 9 through 12, respectively. From these figures. it can be seen that the maximum rarefaction occurs at the counter pole, which is followed by the one situated at $z=-1.67$ in. This one is moderate in magnitude and lasts longer. Figure 12 shows that the opposite pole is under negative pressure (tension) during a great part of the impact period.

As for shear stresses in the viscoelastic core, from the exponential time pulse, there are two regions where the shear stresses are significant. One is located along a line, approximately parallel to and 0.4 in. away from the axis of symmetry. The other one is situated along the circumference of a 2.2 in. radius circle, i.e., approximately 0.8 in. from the surface of the shell. The shear stresses in these two regions are given in figures 13 and 14 respectively. In figure 13 the four curves represent the shear stresses at times t = 0.16, 0.56, 2.48, and 3.34 msec, respectively. While the three curves in figure 14 show the shear stress at times $t = 0.18$, 0.62, and 1.42 msec, respectively. The maximum shear stress is approximately 1.2 lb/in.^{2}. With shear stress of this magnitude, these two regions are possible locations of brain damage, according to the rational hypothesis.

The stress distributions in the shell are shown in Figure 15. The three curves there represent the normal stress in the polar angle direction at times $t = 0.10$, 1.68, and 2.72 msec, respectively. From these it can be seen that stresses at the two poles are relatively large. It is generally believed that skull fractures are due to high tensile stresses; and in view of this, a skull fracture would probably occur at either pole or in the neighborhood of a cone with an extended angle of 50°. Further, both locations also have high shear stresses.

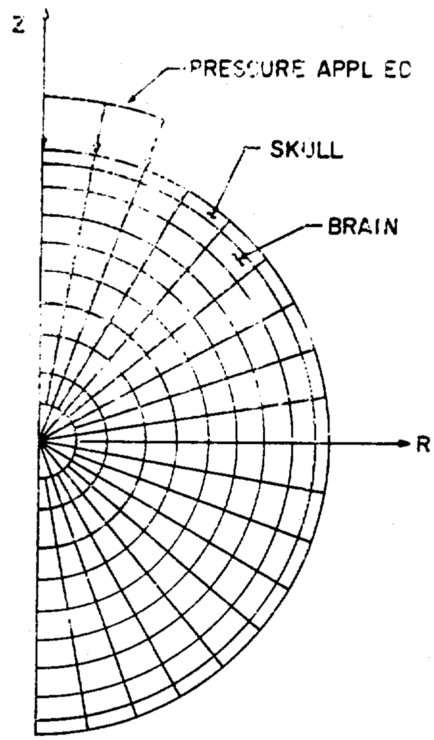

Figure 1. Finite Element Mesh

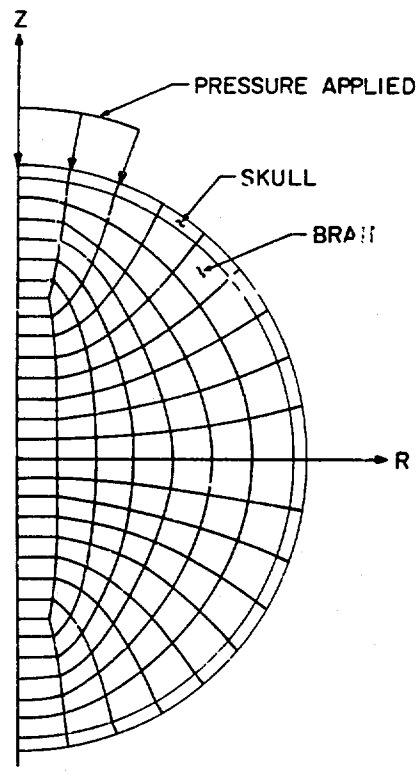

Figure 2. Finite Element Mesh

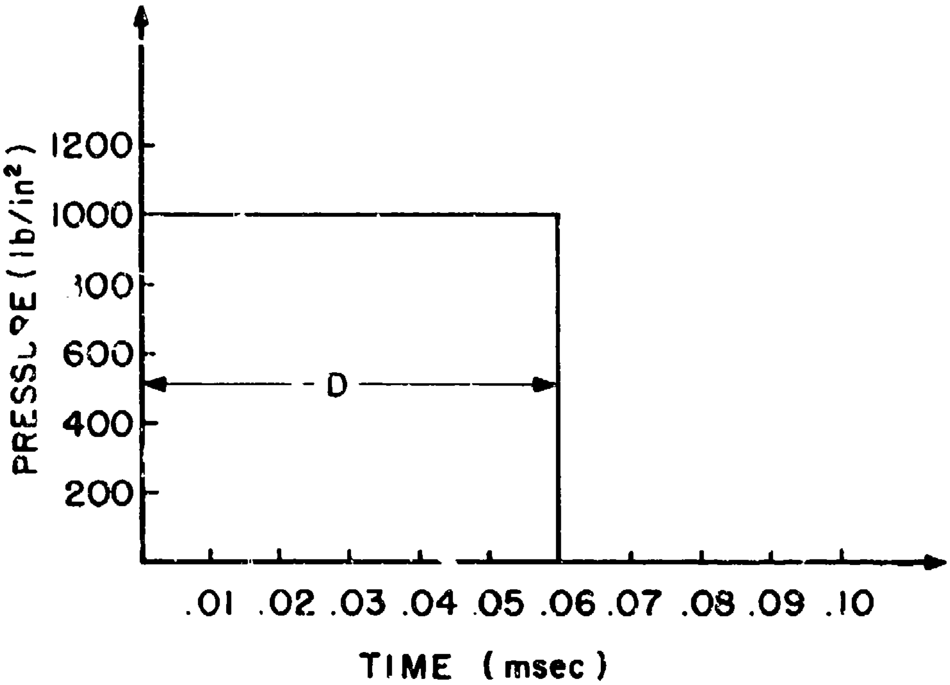

Figure 3. Square Input Pulse

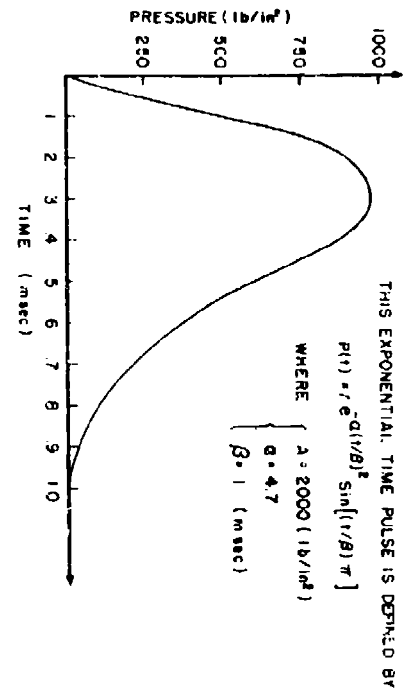

Figure 4. Exponential Input Pulse

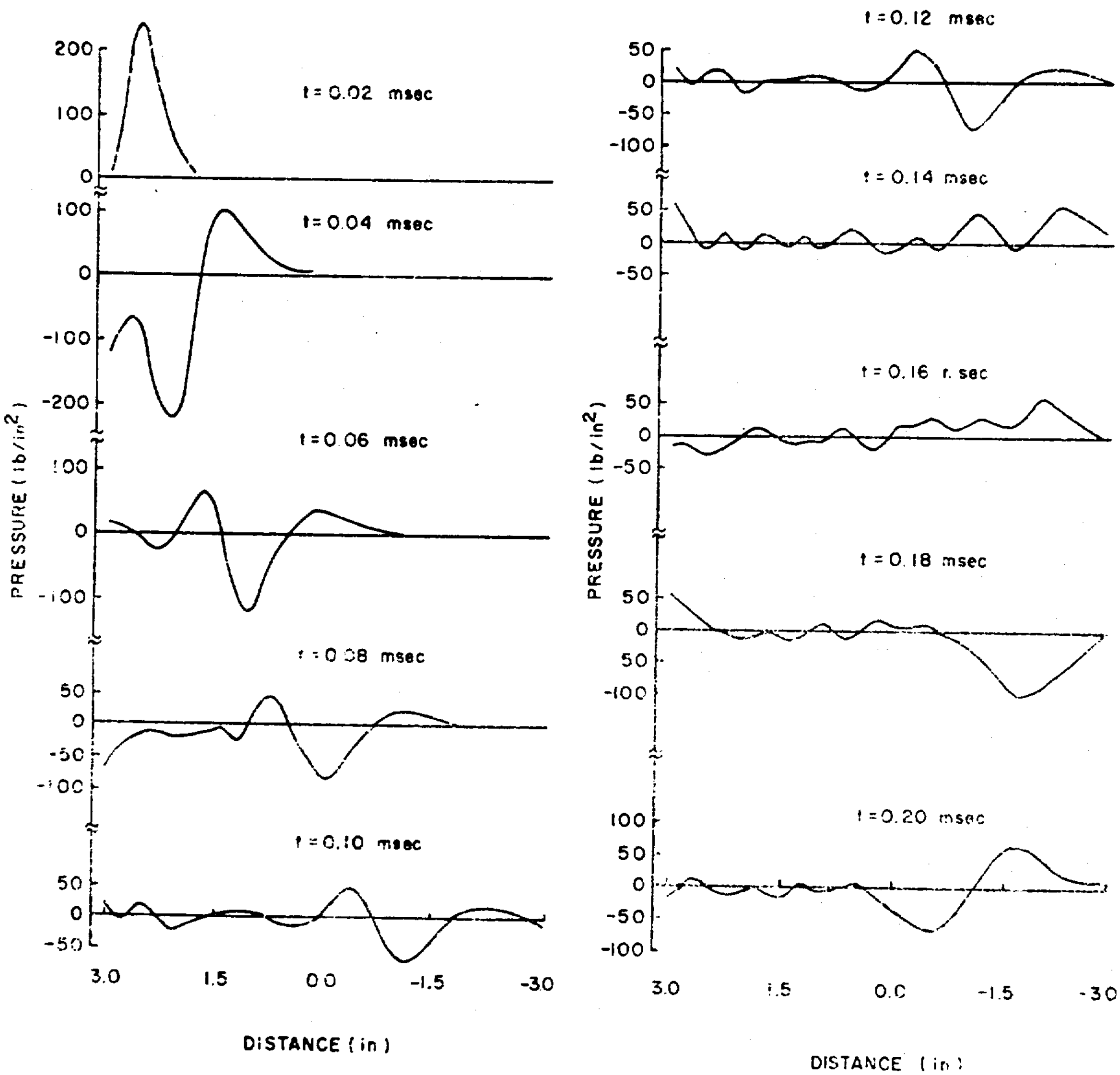

Figure 5. Pressure History Along the Axis of Symmetry for Square Pulse of 0.01 msec Duration

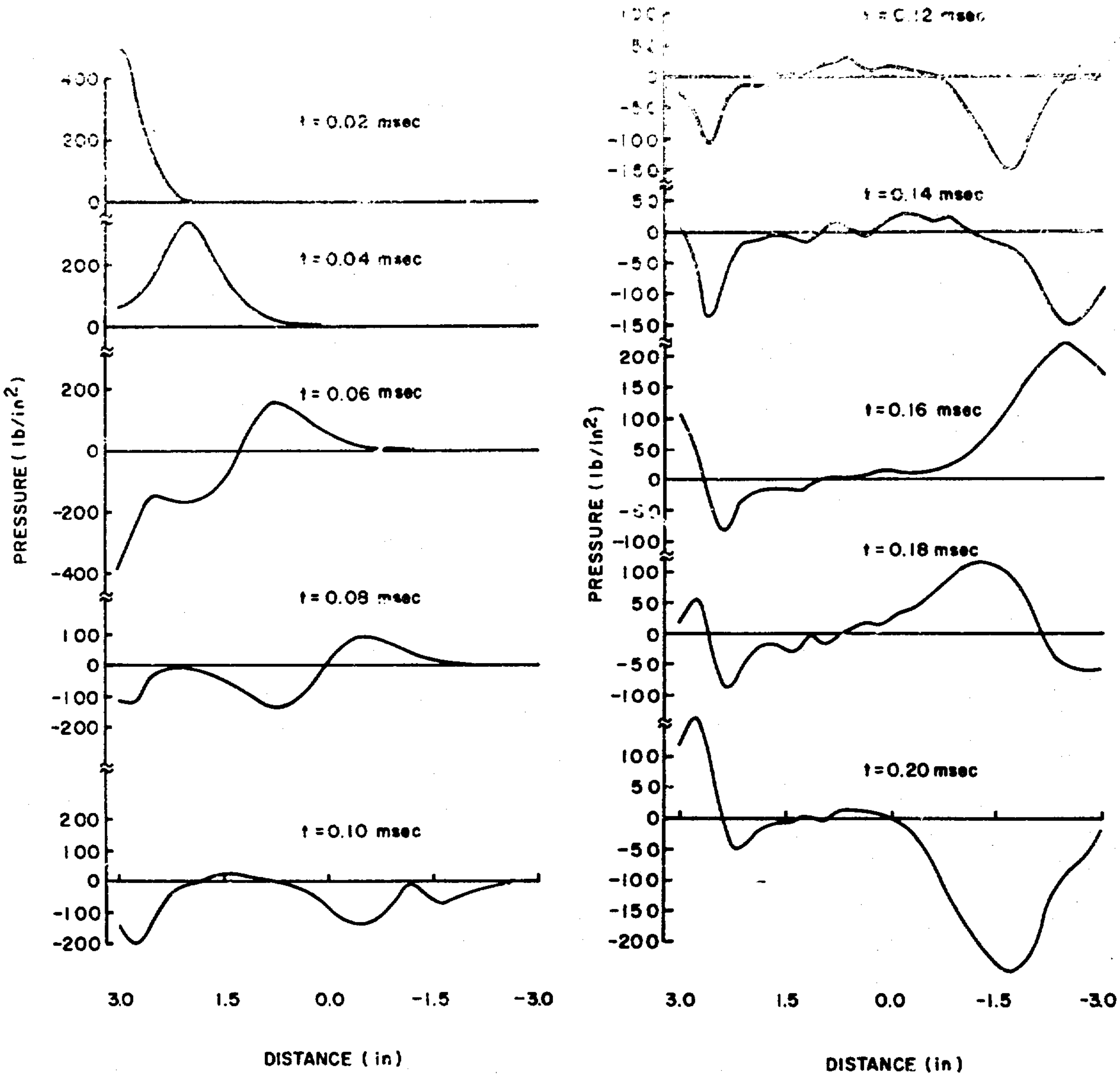

Figure 6. Pressure History Along the Axis of Symmetry for Square Pulse of 0.06 msec Duration

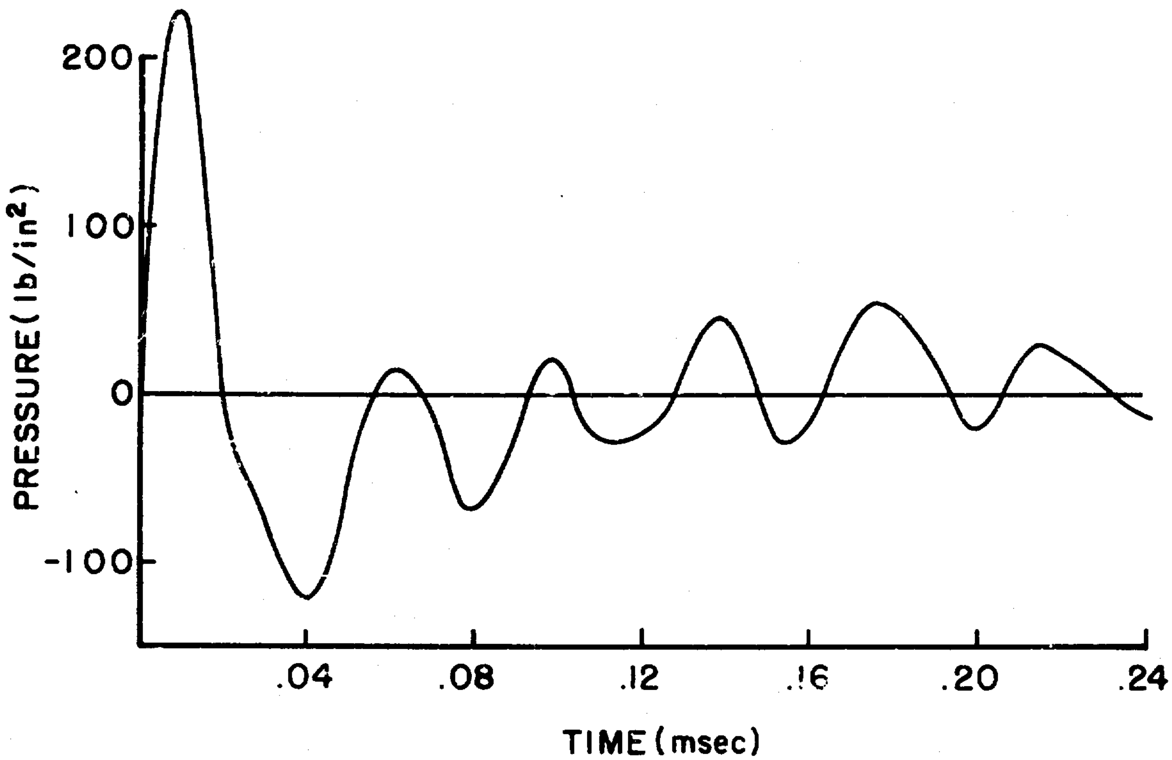

Figure 7. Pressure of Brain at the Impact Pole for Square Pulse of .01 msec Duration

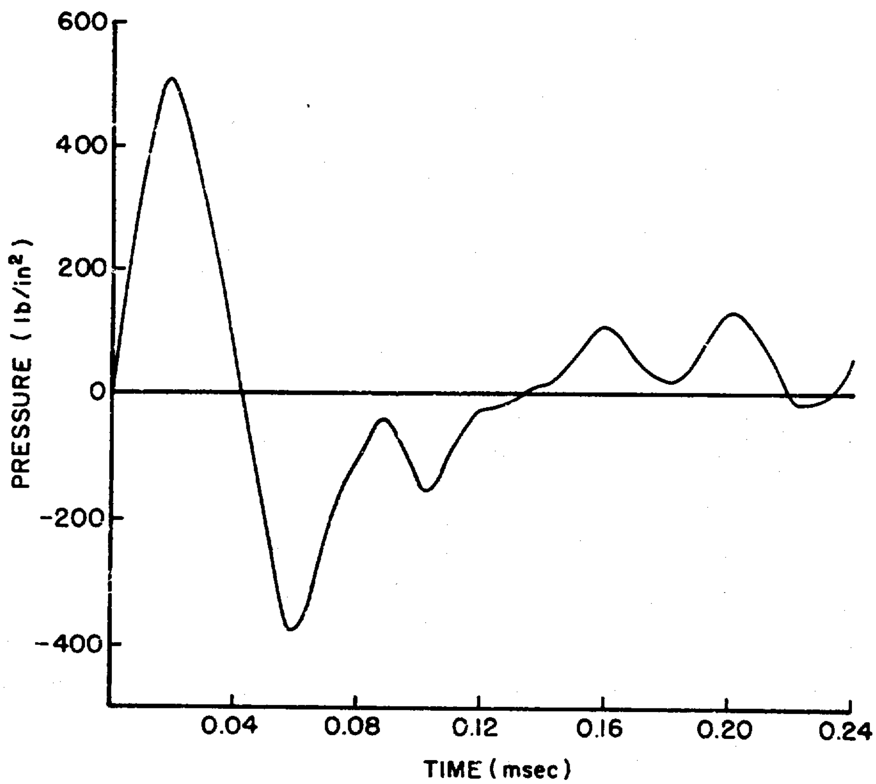

Figure 8. Pressure of Brain at the Impact Pole for Square Pulse of 0.06 msec Duration

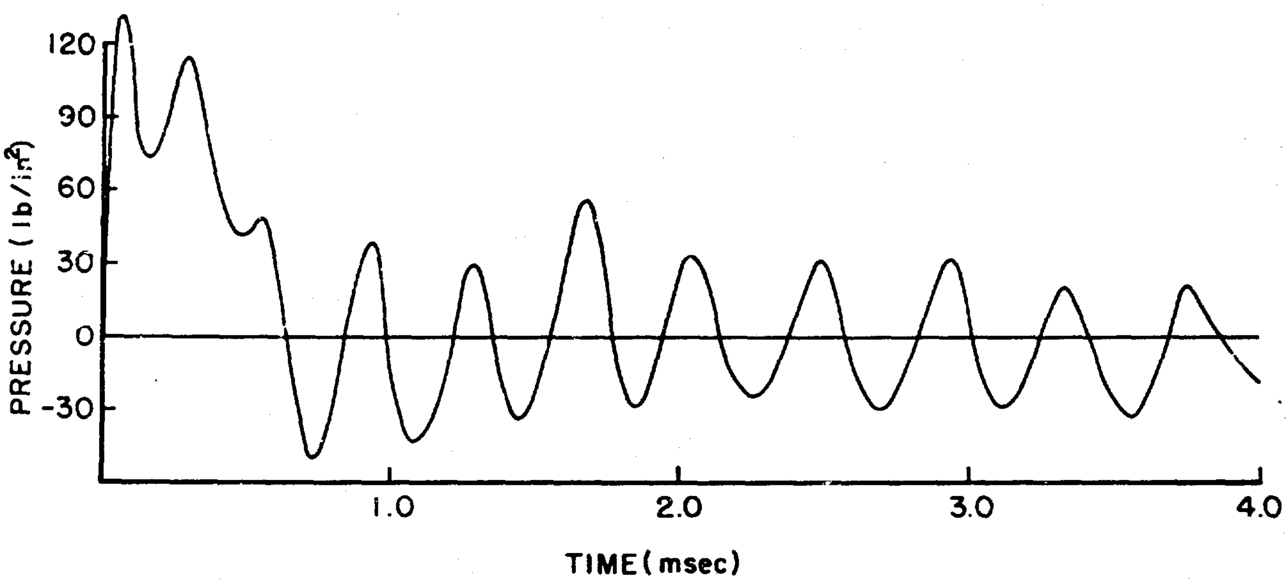

Figure 9. Pressure History at the Impact Pole for Exponential Time Pulse

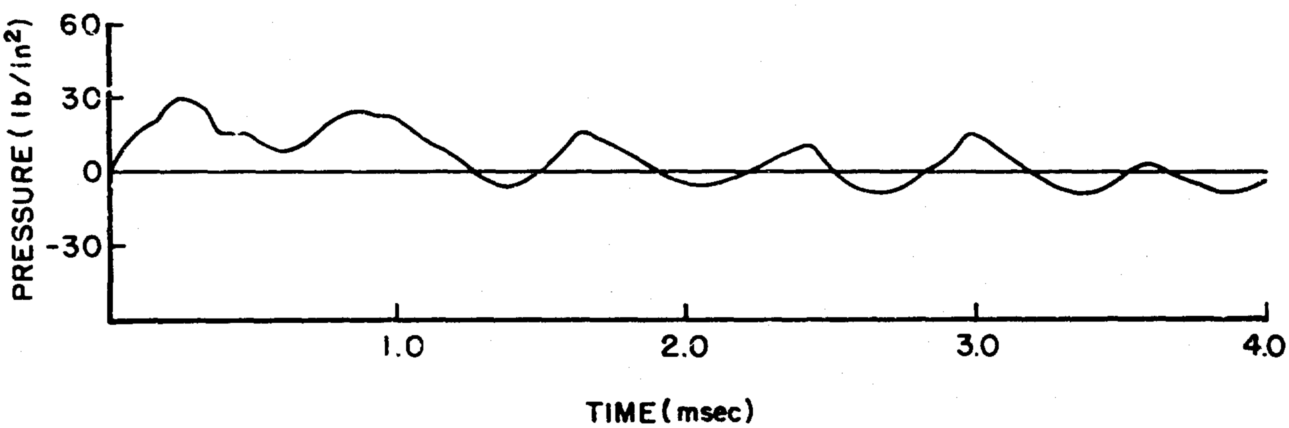

Figure 10. Pressure History Along the Diametric Axis at $z = 0.12$ in for Exponential Time Pulse

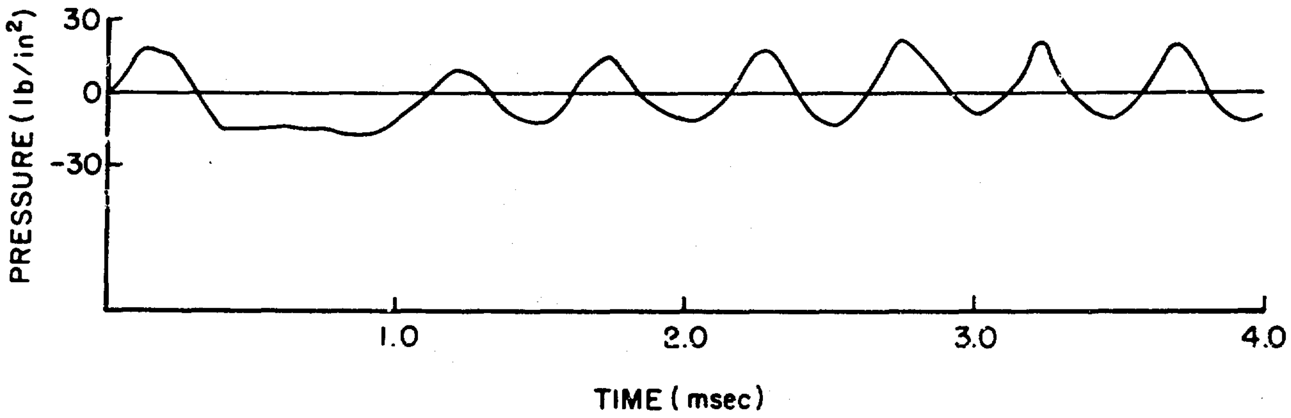

Figure 11. Pressure History at $z = -1.67$ in. on the Diametric Axis for Exponential Time Pulse

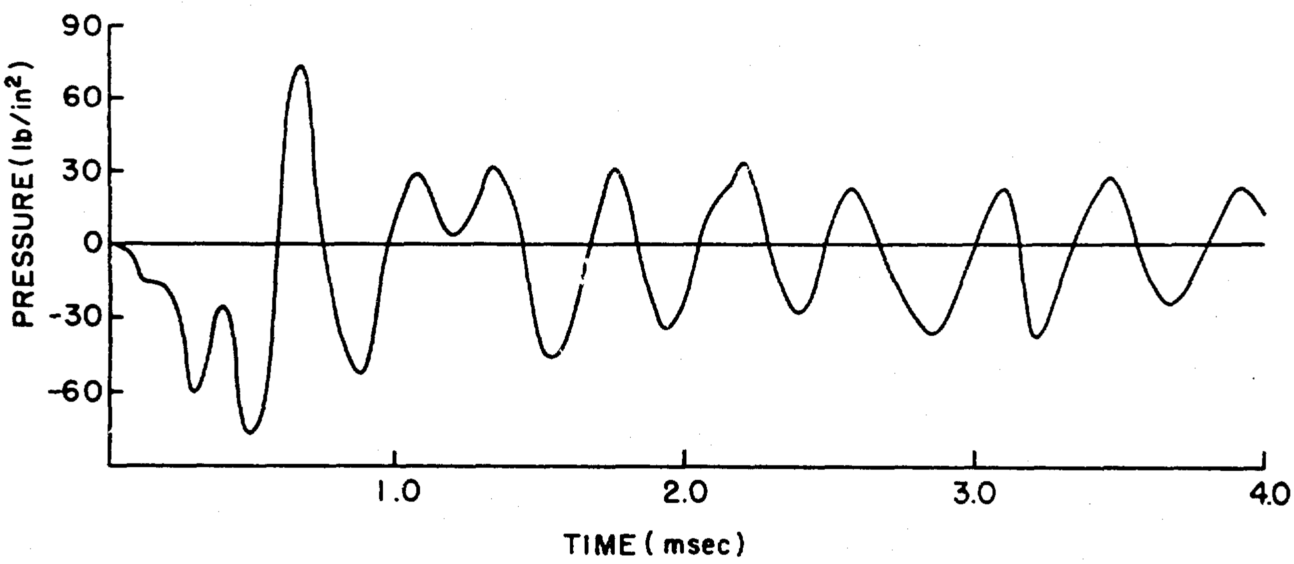

Figure 12. Pressure History at the Opposite Pole for Exponential Time Pulse

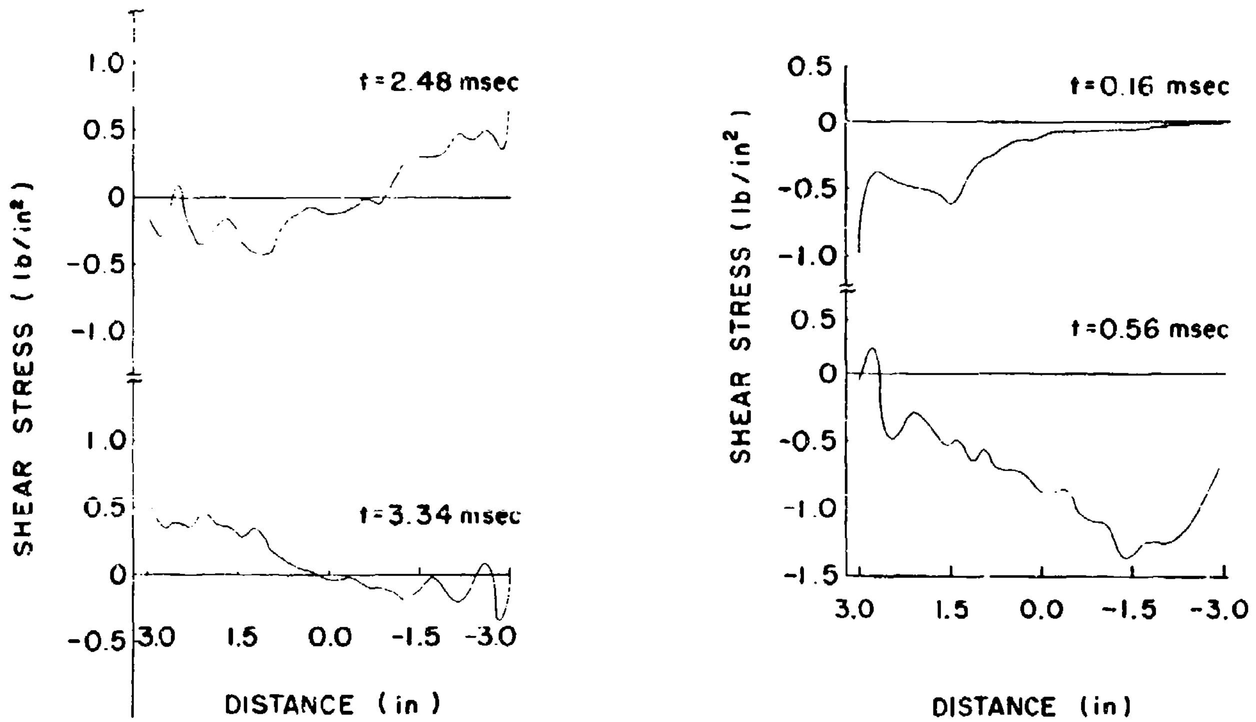

Figure 13. Shear Stress History Along the Diametric Axis for the Exponential Time Input Pulse

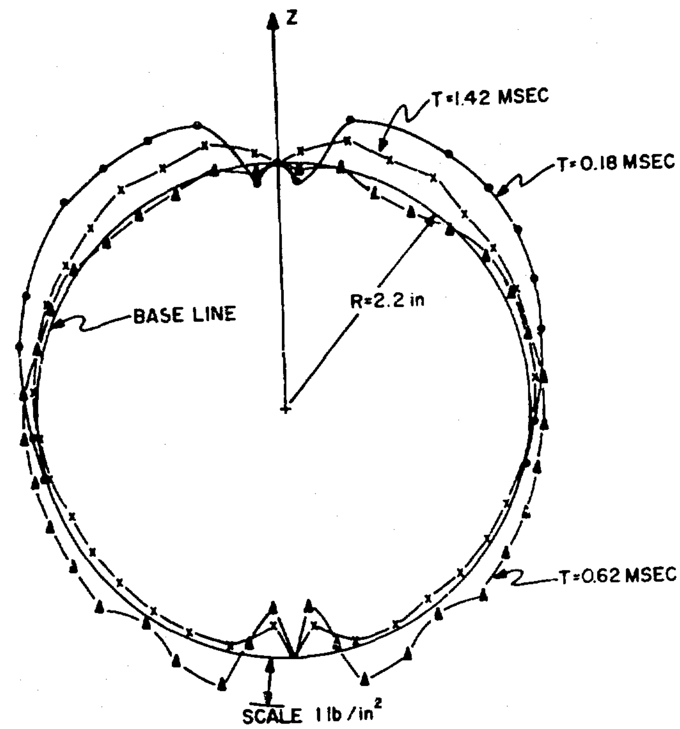

Figure 14. Shear Stress of Brain at a Radius of 2.2 in at Various Times

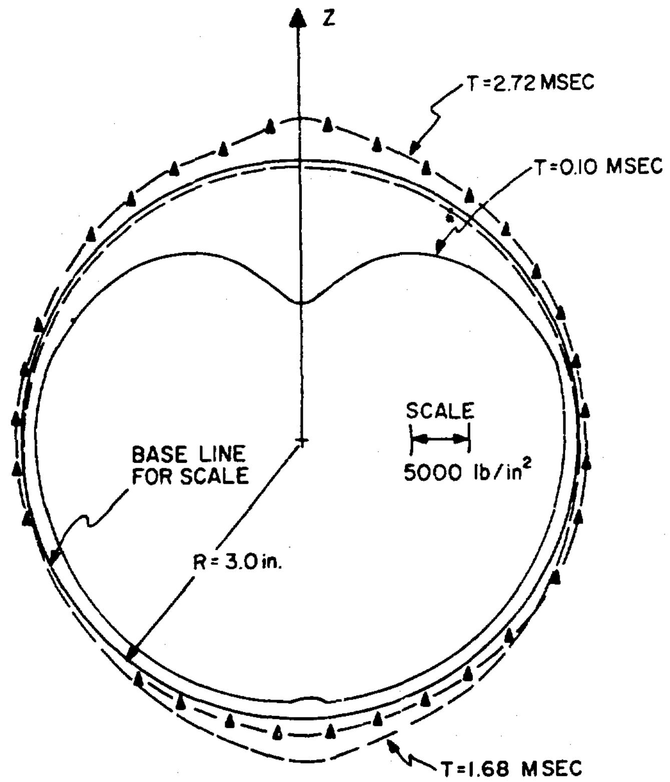

Figure 15. Normal Stress in the Skull

REFERENCES

1. Anon., "A Survey of Current Head Injury Research" Subcommittee on Head Injury, National Advisory Council on Neurological Disease and Stroke, 1969.

2. Anzelius, A., "The Effect of an Impact on a Spherical Liquid Mass." *Acta. Path. Microbiol. Scand.* Supplement 48, 1943.

3. Chan, H. S., *The Asymmetric Response of a Fluid-Filled Spherical Shell—a Mathematical Simulation of Head Injury*. Ph.D. dissertation, Tulane University, Nov., 1971.

4. Clough, R. W. and Rashid, Y. (1965) "Finite Element Analysis of Axisymmetric Solids" *Proc. A.S.C.E.*, 91, EM. 1, p. 71.

5. Engin, A. E., "Axisymmetric Response of a Fluid-Filled Spherical Shell to a Local Radial Impulse—a Model for Head Injury" *J. of Biomechanics*, Vol. 2, No. 3, 325, July, 1969.

6. Engin, A. E. and Liu, Y. K., "Axisymmetric Response of a Fluid-Filled Spherical Shell in Free Vibrations" *J. of Biomechanics*, Vol. 3, No. 1, 11, January, 1970.

7. Engin, A. E. and Roberts, V. L. (1970) "A Mathematical Model for the Behavior of the Brain When the Human Head is Subjected to Impulsive Loads" *Proc. Biodynamics Modelling & Its Applications*, Dayton, Ohio, pp. 877–903.

8. Goldsmith, W., "The Physical Process Producing Head Injury" *Proc. Head Injury Conf.* Lippincott, pp. 350–382, 1966.

9. Guttinger, W. "Der Stosseffect auf eine Flüssigkeitskugel als Grundlage einer Physikalischen Theorie der Entstehung von Gehirnverletzungen," *Zeit. f. Naturforschung*, 5, 622, 1950.

10. Hickling, R. and Wenner, M. L. (1972) *Mathematical Model of a Head Subjected to an Axisymmetric Impact* GMR–1241, General Motors Research Lab., Warren, Michigan.

11. Holbourn, A. H. S., "Mechanics of Head Injury" *Lancet*, 11, p. 438, 1943.

12. Lee, Y. C. and Advani, S. H., "Forced Axisymmetric Response of a Fluid-Filled Spherical Shell," to appear in *5th Southeast Conf. on Theor. & Appl. Mech.*, 1970.

13. Oestreicher, H. L., "Field and Impedance of an Oscillating Sphere in a Viscoelastic Medium with an Application to Biophysics" *J. of The Acoustical Society of America*, Vol. 23, Nov. 1951.

14. Ommaya, A. K., Hirsh, A. E. and Martinez, J., "The Role of Whiplash in Cerebral Concussion," *Proc. Tenth Stapp Car Crash Conf.*, New York: Soc. Automotive Engineering, p. 197–203, 1966.

15. Owings, R. P. (1971) *Forced Axisymmetric Response of a Spherical Shell with an Elastic Core* pending Ph.D. dissertation, West Virginia Univ., Morgantown, W. Va. Private communications from Prof. S. H. Advani.

16. von Gierke, H. E., "On the Dynamics of Some Head Injury Mechanisms" *Biodynamics* and Bionics Division, Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio.

17. Zienkiewicz, O. C., (1967) "The Finite Element Method" McGraw-Hill. New York.

18. Koeneman, J. B. 1966 *Viscoelastic Properties of Brain Tissue.* M.S. thesis, Case Western Reserve University.

19. Fung, Y. C., N. Perrone and M. Anliker, *Biomechanics, Its Foundation and Objectives*. Prentice-Hall, 1972.

☆U.S.Government Printing Office: 1974 — 657-017/363